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General Certificate of Education (A-level)
June 2012

Mathematics MS04

(Specification 6360)

Statistics 4

Mark Scheme

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all examiners participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for standardisation each examiner analyses a number of students' scripts: alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, examiners encounter unusual answers which have not been raised they are required to refer these to the Principal Examiner.

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Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
√or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
$-x$ EE	deduct x marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

MS04

Q	Solution	Marks	Total	Comments
1	H_0: $\mu_1 = \mu_2$ H_1: $\mu_1 \neq \mu_2$	B1		Both
	$\bar{x}_1 = 6.362$ $s_1 = 0.3062$ ($s_1^2 = 0.09377$)	B1		Both (or 0.2887)
	$\bar{x}_2 = 6.548$ $s_2 = 0.2926$ ($s_2^2 = 0.08562$)	B1		Both (or 0.2737)
	$s^2 = \dfrac{8 \times 0.3062^2 + 7 \times 0.2926^2}{9+8-2} = 0.08996$	M1A1		Or: $\dfrac{9 \times 0.2887^2 + 8 \times 0.2737^2}{9+8-2}$
	$t_{calc} = \dfrac{0.1862 - 0}{0.2999\sqrt{\frac{1}{9}+\frac{1}{8}}} = 1.28$	M1A1		awrt
	$\nu = 15$ $t_{crit} = \pm 1.753$	B1B1		Both signs not required.
	$1.28 < 1.753 \Rightarrow$ accept H_0. Insufficient evidence to indicate that means are different.	A1√	10	(Compares) – may be implied; and states conclusion in context. √ t (requires final M1 only).
	Total		**10**	
2(a)	$s = 0.0157762$ …or $s^2 = 0.0002488$...	B1		$\sum (x - \bar{x})^2 = 0.00224$ awrt
	$\chi_9^2(0.01) = 2.088$ $\chi_9^2(0.99) = 21.666$	B1		Both
	$\sqrt{\dfrac{9s^2}{21.666}}$ and $\sqrt{\dfrac{9s^2}{2.088}}$	M1 A1A1		Or variances
	(0.0102 , 0.0328)	A1	6	accept $\geq$ 1sf e.g. (0.01,0.03)
(b)	H_0: $\sigma_A^2 = \sigma_B^2$ H_1: $\sigma_A^2 \neq \sigma_B^2$	B1		Both
	$s_A^2 = 0.0008571$... $s_B^2 = 0.0002488$...	B1		Both
	$F_{calc} = \dfrac{0.0008571...}{0.0002488...} = 3.44$	M1A1		awrt
	$\nu_A = 9$ $\nu_B = 7$, $F_{7,9}(0.10) = 3.293$	B1B1		
	$3.44 > 3.293 \Rightarrow$ Reject H_0 Sufficient evidence to conclude that variances differ.	A1√	7	(Compare) and state conclusion. √ on F and CV.
	Total		**13**	

MS04 (cont)

Q	Solution	Marks	Total	Comments
3(a)	$\bar{x} = \frac{125}{100} = 1.25$	B1		cao
	$5p = 1.25 \Rightarrow p = 0.25$	B1	2	cao
(b)	Distribution B(5,0.25): 0.2373 0.3955 0.2637 0.0879 0.0146 0.0010 (May be implied by E_i s.)	M1A1 A1		A1 for 3 correct. 2nd A1 for all correct.(≥3dp)
	Expected frequencies are: 23.73 39.55 26.37 8.79 1.46 0.1	A1√	4	Probabilities × 100.
(c)	H_0: B(5, p) is an appropriate model.	B1		Condone p = 0.25.
	O 25 41 20 14 **E** 23.73 39.55 26.37 10.35	M1		Combines last three classes.
	$\chi^2_{calc} = \sum \left\{ \frac{(O-E)^2}{E} \right\} = 2.947$	M1A1		awfw 2.94 to 2.95
	$\nu = 4 - 2 = 2$ $\chi^2_{crit} = 5.99$	B1B1		
	$2.947 < 5.99 \Rightarrow$ Accept H_0 B(5, p) is a suitable model.	A1√	7	(Compare) and state conclusion in context. √ on χ^2
(d)	It gives some support. If, for example, the probabilities were different for a seed in the front row, say, then this would not be discernible from figures for 100 rows.	B1√ B1dep	2	Ft on their conclusion
	Total		**15**	

MS04 (cont)

Q	Solution	Marks	Total	Comments
4(a)	Unbiased estimator with the smaller Variance would, more often than not, yield an estimate closer to the parameter.	E2	2	SC 'implies more efficient' E1
(b)(i)	$\mathrm{E}(\overline{X}_1 - \overline{X}_2) = \mathrm{E}(\overline{X}_1) - \mathrm{E}(\overline{X}_2) = \mu_1 - \mu_2$	M1A1		
	$\mathrm{Var}(\overline{X}_1 - \overline{X}_2) = \dfrac{\sigma_1^{\ 2}}{n_1} + \dfrac{\sigma_2^{\ 2}}{n_2}$	B1	3	
(ii)	$V = \dfrac{\sigma_1^{\ 2}}{n_1} + \dfrac{\sigma_2^{\ 2}}{n - n_1}$	M1		Or in terms of n_2 etc
	$\dfrac{\mathrm{d}V}{\mathrm{d}n_1} = -\dfrac{\sigma_1^{\ 2}}{n_1^{\ 2}} + \dfrac{\sigma_2^{\ 2}}{(n-n_1)^2} = 0$	m1A1		
	$\Rightarrow \dfrac{\sigma_1^{\ 2}}{\sigma_2^{\ 2}} = \dfrac{n_1^{\ 2}}{(n-n_1)^2} = \dfrac{n_1^{\ 2}}{n_2^{\ 2}} \Rightarrow \dfrac{\sigma_1}{\sigma_2} = \dfrac{n_1}{n_2}$	A1	4	AG
(iii)	$n_1 : n_2 = \sigma_1 : \sigma_2 = 5:9 \quad (\Rightarrow 14\text{parts})$	M1		
	$n_1 = 5 \times 20 = 100 \quad n_2 = 9 \times 20 = 180$	A1	2	Both cao
	Total		**11**	

MS04 (cont)

Q	Solution	Marks	Total	Comments
5(a)	$\mathrm{E}(X^2)=\int_0^\infty kx^2\mathrm{e}^{-kx}\,\mathrm{d}x$	M1		
	$=\left[-x^2\mathrm{e}^{-kx}\right]_0^\infty+\int_0^\infty 2x\mathrm{e}^{-kx}\,\mathrm{d}x$	M1		
	$=0+\left[-\frac{2x}{k}\mathrm{e}^{-kx}\right]_0^\infty+\int_0^\infty \frac{2}{k}\mathrm{e}^{-kx}\,\mathrm{d}x$	A1		0 may be omitted, or limits inserted at end of process. (E(X) integral can be quoted.)
	$=0+\left[-\frac{2}{k^2}\mathrm{e}^{-kx}\right]_0^\infty$	A1		Ditto.
	$=\frac{2}{k^2}$	A1		
	$\mathrm{Var}(X)=\frac{2}{k^2}-\left(\frac{1}{k}\right)^2=\frac{1}{k^2}$	A1√	6	Their E(X^2) minus mean^2, provided **positive**. **SC** Allow B1 for those who write correct working and result, having failed to integrate correctly.
(b)(i)	$\mathrm{F}(x)=\int_0^x k\mathrm{e}^{-ku}\,\mathrm{d}u$	M1		
	$=\left[-\mathrm{e}^{-ku}\right]_0^x \quad =1-\mathrm{e}^{-kx}$	A1A1	3	
(ii)	$\left[1-\mathrm{e}^{-kx}\right]_0^N=0.9\Rightarrow \mathrm{e}^{-kN}=0.1$	M1		
	$\Rightarrow N=\frac{1}{k}\ln 10$	M1A1	3	M1 for taking logs. cao, acf
(c)	Mean = $a=\frac{1}{k}\Rightarrow k=\frac{1}{a}$,			
	Mean = $3a=\frac{1}{k}\Rightarrow k=\frac{1}{3a}$	M1A1		
	$\mathrm{e}^{-\frac{1}{a}\cdot a}\cdot\mathrm{e}^{-\frac{1}{3a}\cdot a}=\mathrm{e}^{-1}\cdot\mathrm{e}^{-\frac{1}{3}}=\mathrm{e}^{-\frac{4}{3}}$	M1A1	4	cwo
	Total		**16**	

MS04 (cont)

Q	Solution	Marks	Total	Comments
6(a)(i)	$E(X) = p + 2qp + 3q^2p + 4q^3p + \ldots$	M1		$q = 1 - p$
	$= p(1 + 2q + 3q^2 + 4q^3 + \ldots)$			
	$= p(1-q)^{-2}$	M1		
	$= \dfrac{p}{p^2} = p^{-1}$	A1	3	
(ii)	$\text{Var}(X) = \dfrac{2-p}{p^2} - \left(\dfrac{1}{p}\right)^2$	M1		
	$= \dfrac{2-p-1}{p^2} = \dfrac{1-p}{p^2}$	A1	2	$\left(\text{or } \dfrac{q}{p^2}\right)$
(b)(i)	P(> 2 throws required) $= 1 - \left(\dfrac{1}{4} + \dfrac{3}{4} \times \dfrac{1}{4}\right) = \dfrac{9}{16}$	M1A1	2	
(ii)	$E(Y) = E(1+X) = 1 + E(X) = 1 + \dfrac{4}{3} = \dfrac{7}{3}$	B1		(OE)
	$\text{Var}(Y) = \text{Var}(1+X)$ $= 0 + \text{Var}(X) = \dfrac{\left(\frac{1}{4}\right)}{\left(\frac{3}{4}\right)^2} = \dfrac{4}{9}$	M1A1	3	(OE)
	Total		**10**	
	TOTAL		**75**	